On Possibility of Decoherence
in Correlated two Neutral Kaon Systems

Fumiyo Uchiyama[1,2]

[1] *Physics Division, Lawrence Berkeley National Laboratory, Berkeley, CA 94720, USA*
[2] *Department of Physics, Science University of Tokyo, Noda, Chiba, Japan*

Abstract

We point out that decoherence parameters in the correlated two neutral kaon system breaks the transformation invariance of basis and their magnitude are stringently limited by the experimentally measured magnitudes of CP violation and of the strangeness non conserving $\Delta S = \pm 2$ transitions.

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I. INTRODUCTION

It has been long since the neutral kaons played an important role in the history of quantum mechanics showing that the particle mixture [1] exhibit unusual and peculiar properties, the oscillating behaviors in the probabilities of finding a K^0 (or a $\bar{K}^0$) in a beam which is initially pure $\bar{K}^0$ (or K^0), as function of time [2]. This property of producing other particles stems from the fact that the strangeness eigenstates, K^0, $\bar{K}^0$ are different from eigenstates of Hamiltonian, K_S and K_L. The fact that the inverse of their mass difference is the order of one of their life times made possible for the oscillation to be seen in the laboratory. The oscillations in the probabilities stem from the interference terms in the particle mixture states coming from quantum mechanical superposition principle. Based upon these striking quantum mechanical features of kaons, correlated two neutral kaons have been used in the proposals for testing EPR [3] against Quantum Mechanics [4].

Eberhard introduced a decoherence parameter ζ which parameterizes the deviation from Quantum Mechanics in his proposals [6] on non-locality experiment. The experiment involves measurements of K_S and/or K_L particle states from decays of Φ mesons; explicitly, in the analysis for the proposed experiment, the probability of finding two kaons in states, f_1 and f_2, in the decay products of Φ mesons, is written as

$$
\begin{aligned}
P_{decoh}(f_1, f_2) = \frac{1}{2}[&|\langle f_1|K_S\rangle\langle f_2|K_L\rangle|^2 + |\langle f_1|K_L\rangle\langle f_2|K_S\rangle|^2 \\
&-(1-\zeta)(\langle f_1|K_S\rangle\langle f_2|K_L\rangle\langle f_1|K_L\rangle^*\langle f_2|K_S\rangle^* \\
&+\langle f_1|K_L\rangle\langle f_2|K_S\rangle\langle f_1|K_S\rangle^*\langle f_2|K_L\rangle^*)]
\end{aligned} \tag{1}
$$

At $\zeta = 0$, $P_{decoh}(f_1, f_2)$ agrees with the result from a quantum mechanical calculation while a nonzero decoherence parameter indicates an existence of a deviation from quantum mechanics. As the extreme case, $\zeta = 1$ gives zero interference terms in the above expression. Furry [7] in 1936, before the discovery of kaons, discussed theoretically in detail the degrees of agreement and disagreement between the results of quantum mechanical calculations and those to be expected on the assumption that a system once freed from dynamical interference (corresponding to $\zeta = 1$ in the case of two correlated kaons) can be regarded as possessing independently real properties. Recently the decoherence parameters in the interference terms in two neutral kaons, K^0 and $\bar{K}^0$, are studied [5] in terms of asymmetry in probabilities for like and unlike strangeness events using equation (1).

The interference terms in probabilities are considered to carry quantum mechanical features and indeed such are the cases for the superposition of two waves of one electron passing different routes in space-time and coming back together later, and no interference in its superposed wave functions means that the system can be treated as " classical", namely particle-like. Interestingly, it will become clear in this paper that the interference terms in two correlated particle mixture do not have this significance.

High energy accelerators made it possible to produce heavy flavored neutral mesons such as D and B, for which similar oscillations could be observed and neutrino oscillations among different flavored massive neutrinos, are predicted and many experimental investigation are underway. Considering these facts, elucidation of the limitation and meaningfulness of decoherence (separability in other language) parameters in correlated two particle systems is in urgent need. It is also important in the fields of quantum information theory [8].

The purposes of this paper are two: (1) to show that deduction of the magnitude of inter-ference terms for the case of mixture of particles, even for the case of complete spontaneous factorization($\zeta = 1$) taking place does not imply that the system is "separable", namely "more classical", contrary to our intuition learned from the superposition of wave functions in space though it will serve as an indicator of deviation from quantum mechanics whenever ζ is nonzero. (2) to show explicitly that the magnitude of decoherence parameters are lim-ited by the magnitude of CP violation, and the magnitudes of strangeness non-conserving $|\Delta S| = 2$ transitions, and to show smallness of decoherence parameters leads to the ne-cessity for introducing a new decoherence parameters for each pair of different eigenstate measurements if an expression like equation (1) is desirable to be used for analysis.

II. FORMULATION

We use mass and strangeness eigenstates (K_S, K_L) and $(K^0, \bar{K}^0)$ as the two sets of bases, which are more frequently used eigenstates in experiments rather than two arbitrarily chosen sets. This particular choice of sets enable us to derive the results by clear and definite arguments . The latter set is an orthogonal bases set while the former set is not. It can be easily seen at the end of this section that we can develop our arguments more generally , using any sets of two independent basis vectors.

The bases in the two sets are related at <u>all</u> time

$$|K_S\rangle = \frac{N}{\sqrt{2}}[(1+\epsilon)|K^0\rangle - (1-\epsilon)|\bar{K}^0\rangle] \tag{2}$$

$$|K_L\rangle = \frac{N}{\sqrt{2}}[(1+\epsilon)|K^0\rangle + (1-\epsilon)|\bar{K}^0\rangle] \tag{3}$$

where ϵ is the CP violation parameter ($|\epsilon| \sim 10^{-3}$) and N is a normalization constant, $\frac{1}{\sqrt{(1+|\epsilon|^2)}}$.

We consider the physical situations in which two neutral kaons are produced in correlated states of $J^{PC} = 1^{--}$ due to ϕ mesons decay. At time t after the ϕ decays, the two kaon state is expressed in the strangeness basis $(K^0, \bar{K}^0)$ as

$$\frac{1}{\sqrt{2}}(|K_0(t)\rangle_r |\bar{K}_0(t)\rangle_l - |\bar{K}_0(t)\rangle_r |K_0(t)\rangle_l) \tag{4}$$

where r and l stand for left and right to distinguish the two kaons. Or similarly in (K_S, K_L), the two kaon state is,

$$\frac{N'}{\sqrt{2}}(|K_S(t)\rangle_r |K_L(t)\rangle_l - |K_L(t)\rangle_r |K_S(t)\rangle_l) \tag{5}$$

where $N' = \frac{(1+|\epsilon|^2)}{(1-\epsilon^2)} = 1 + O(|\epsilon|^2)$ and

$$|K_{S,L}(t)\rangle = e^{-\frac{i}{2}\lambda_{S,L}}|K_{S,L}(0)\rangle$$

where $\lambda_{S,L}$ are complex mass eigenvalues of K_S and K_L respectively.

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The probability of finding an f_1-type kaon to the right and an f_2-type kaon to the left at time t in the (K_S, K_L) basis is given by

$$P(f_1, f_2; t) = \frac{1}{2}|\langle f_1|K_S\rangle_r \langle f_2|K_L\rangle_l - \langle f_1|K_L\rangle_r \langle f_2|K_S\rangle_l|^2 + O(\epsilon^2) \qquad (6)$$

where time t is implicit in kaon states and the correction from the normalization of order $O(\epsilon^2)$ is separately written. Similarly the probability expressed in $(K^0, \bar{K}^0)$ bases, is given by

$$P(f_1, f_2, t) = \frac{1}{2}|\langle f_1|K_0\rangle\langle f_2|\bar{K}_0\rangle - \langle f_1|\bar{K}_0\rangle\langle f_2|K_0\rangle|^2 \qquad (7)$$

where we have omitted r and l subscripts with the understanding that the first and second states in pairwise have r and then l subscripts respectively. Here we make further simplification taking $\epsilon = 0$. This choice makes it clear that the derivation of our conclusions is independent of weak interactions. To make our conclusions applicable to the mass eigenstates,(K_S, K_L), we will consider corrections later by turning on weak interactions. The corrections are perturbative and order of ϵ. Substituting $\epsilon = 0$ into equations (2, 3), we get two CP eigenstates,

$$|K_S\rangle_{\epsilon=0} = \frac{1}{\sqrt{2}}[|K^0\rangle - |\bar{K}^0\rangle]$$

$$|K_L\rangle_{\epsilon=0} = \frac{1}{\sqrt{2}}[|K^0\rangle + |\bar{K}^0\rangle] \qquad (8)$$

which form another orthogonal basis set. In this basis, the two kaons from a Φ decays can be expressed as

$$\frac{1}{\sqrt{2}}(|K_S(t)\rangle_{\epsilon=0}|K_L(t)\rangle_{\epsilon=0} - |K_L(t)\rangle_{\epsilon=0}|K_S(t)\rangle_{\epsilon=0}) \qquad (9)$$

The probability in this basis is given by

$$P(f_1, f_2; t) = \frac{1}{2}|\langle f_1|K_S\rangle_{\epsilon=0}\langle f_2|K_L\rangle_{\epsilon=0} - \langle f_1|K_L\rangle_{\epsilon=0}\langle f_2|K_S\rangle_{\epsilon=0}|^2 \qquad (10)$$

The two sets of orthogonal bases to be used from now on are , $(K_S, K_L)_{\epsilon=0}$ and $(K^0, \bar{K}^0)$ which are eigenstates of CP and Strangeness respectively. They are eigenstates of the Hamiltonian,

$$H = H_{st} + H_{em}$$

where H_{st} and H_{em} are strong and electro-magnetic interaction hamiltonians respectively. Hamiltonian, strangeness and CP operators satisfy commutation relations;

$$[H, CP] = 0; \qquad [H, S] = 0; \qquad [S, CP] \neq 0$$

As it is rather annoying to keep the subscripts $>_{\epsilon=0}$, unless otherwise stated, we use the same notations, K_S and K_L, for CP+ and CP- eigenstates without subscript $_{\epsilon=0}$ now on.

Expanding the terms in the equation (10), the probability is given as

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$$P(f_1, f_2, t) = \frac{1}{2}(|\langle f_1|K_S\rangle\langle f_2|K_L\rangle|^2 + |\langle f_1|K_L\rangle\langle f_2|K_S\rangle|^2$$
$$-\langle f_1|K_S\rangle\langle f_2|K_L\rangle\langle f_1|K_L\rangle^*\langle f_2|K_S\rangle^*$$
$$-\langle f_1|K_L\rangle\langle f_2|K_S\rangle\langle f_1|K_S\rangle^*\langle f_2|K_L\rangle^*) \tag{11}$$

Or equivalently,

$$P(f_1, f_2; t) = \frac{1}{2}[|\langle f_1|K_S\rangle\langle f_2|K_L\rangle|^2 + |\langle f_1|K_L\rangle\langle f_2|K_S\rangle|^2$$
$$-2Re(\langle f_1|K_S\rangle\langle f_2|K_L\rangle\langle f_1|K_L\rangle^*\langle f_2|K_S\rangle^*)] \tag{12}$$

To transform the probability to the one expressed in the set of bases $(K_0, \bar{K}_0)$, we substitute (8) into (11). For the first term in (11), we obtain by direct substitution,

$$|\langle f_1|K_S\rangle\langle f_2|K_L\rangle|^2 = \frac{1}{2}[\langle f_1|K_0\rangle\langle f_2|K_0\rangle - \langle f_1|\bar{K}_0\rangle\langle f_2|\bar{K}_0\rangle + \langle f_1|K_0\rangle\langle f_2|\bar{K}_0\rangle - \langle f_1|\bar{K}_0\rangle\langle f_2|K_0\rangle]$$
$$\cdot[\langle f_1|K_0\rangle^*\langle f_2|K_0\rangle^* - \langle f_1|\bar{K}_0\rangle^*\langle f_2|\bar{K}_0\rangle^* + \langle f_1|K_0\rangle^*\langle f_2|\bar{K}_0\rangle^*$$
$$-\langle f_1|\bar{K}_0\rangle^*\langle f_2|K_0\rangle^*] \tag{13}$$

In order to make calculation transparent, we define the following quantities;

$$A \equiv \langle f_1|K_0\rangle\langle f_2|K_0\rangle - \langle f_1|\bar{K}_0\rangle\langle f_2|\bar{K}_0\rangle \tag{14}$$
$$B \equiv \langle f_1|K_0\rangle\langle f_2|\bar{K}_0\rangle - \langle f_1|\bar{K}_0\rangle\langle f_2|K_0\rangle \tag{15}$$

Note that A has amplitudes of $\Delta S = 2$ and $\Delta S = -2$ transitions while B is strangeness conserving amplitudes. Then equation (13) becomes

$$|\langle f_1|K_S\rangle\langle f_2|K_L\rangle|^2 = \frac{1}{4}(A + B)(A^* + B^*) \tag{16}$$

Similar substitutions into the rest of terms in equation (11) in this bases yields

$$|\langle f_1|K_L\rangle\langle f_2|K_S\rangle|^2 = \frac{1}{4}(A - B)(A^* - B^*)$$
$$\langle f_1|K_S\rangle\langle f_2|K_L\rangle\langle f_1|K_L\rangle^*\langle f_2|K_S\rangle^* = \frac{1}{4}(A + B)(A^* - B^*)$$
$$\langle f_1|K_L\rangle\langle f_2|K_S\rangle\langle f_1|K_S\rangle^*\langle f_2|K_L\rangle^* = \frac{1}{4}(A - B)(A^* + B^*) \tag{17}$$

Summing all these terms up, we obtain for transformed probability in $(K_0, \bar{K}_0)$ bases:

$$P(f_1, f_2, t) = \frac{1}{2}BB^* = \frac{1}{2}|B|^2 \tag{18}$$

which is nothing but the equation(7), the expression in the bases of $(K^0, \bar{K}^0)$ showing the invariance of the probability $P(f_1, f_2; t)$ under base-transformations.

III. INTRODUCTION OF DECOHERENCE PARAMETER

Following Eberhard [6], let's assume that the physical processes of a particular interest might deviate from QM for some unknown reasons and that one of the conceivable parameterization for the probability in a particular experiment involving measurement on pairs of kaons in mass eigenstates K_S and K_L is given by a modification of the interference term as

$$P_{decoh}(f_1, f_2; t) = \frac{1}{2}[|\langle f_1|K_S\rangle\langle f_2|K_L\rangle|^2 + |\langle f_1|K_L\rangle\langle f_2|K_S\rangle|^2$$
$$-(1-\zeta)(\langle f_1|K_S\rangle\langle f_2|K_L\rangle\langle f_1|K_L\rangle^*\langle f_2|K_S\rangle^*$$
$$+\langle f_1|K_L\rangle\langle f_2|K_S\rangle\langle f_1|K_S\rangle^*\langle f_2|K_L\rangle^*)] \tag{19}$$

where ζ is called decoherence parameter [6]. We rewrite $P_{decoh}(f_1, f_2; t)$ above in the $(K^0, \bar{K}^0)$ basis.

Following the steps we took in the previous section for transforming bases and using equations (16) and (17), we obtain

$$P_{decoh}(f_1, f_2; t) = \frac{1}{2}BB^* + (\frac{\zeta}{2})(AA^* - BB^*) \tag{20}$$

which can be explicitly written down in the $(K^0, \bar{K}^0)$ basis as

$$P_{decoh}(f_1, f_2; t) = \frac{1}{2}[|\langle f_1|K_0\rangle\langle f_2|\bar{K}_0\rangle|^2 + |\langle f_1|\bar{K}_0\rangle\langle f_2|K_0\rangle|^2$$
$$-(1-\zeta)(\langle f_1|K_0\rangle\langle f_2|\bar{K}_0\rangle\langle f_1|\bar{K}_0\rangle^*\langle f_2|K_0\rangle^*$$
$$+\langle f_1|\bar{K}_0\rangle\langle f_2|K_0\rangle\langle f_1|K_0\rangle^*\langle f_2|\bar{K}_0\rangle^*) + \zeta \cdot T_{extra}] \tag{21}$$

$\zeta = 0$ reproduces the eq.(7) as we expect. Because of the nonzero term, T_{extra}, the form of $P_{decoh}(f_1, f_2; t)$ becomes no-invariant under the transformation of bases. There is no apriori reason for invariance of the form of $P_{decoh}(f_1, f_2; t)$ under base transformations . (However we will see that the invariance requirement is important for spin -correlated two lepton case at the end of section IV.) The explicit terms in T_{extra} are

$$\frac{1}{2}(|\langle f_1|K_0\rangle\langle f_2| K_0\rangle|^2 + |\langle f_1|\bar{K}_0\rangle\langle f_2|\bar{K}_0\rangle|^2$$
$$-\langle f_1|K_0\rangle\langle f_2|K_0\rangle\langle f_1|\bar{K}_0\rangle^*\langle f_2|\bar{K}_0\rangle^* - \langle f_1|K_0\rangle^*\langle f_2|K_0\rangle^*\langle f_1|\bar{K}_0\rangle\langle f_2|\bar{K}_0\rangle$$
$$-|\langle f_1|K_0\rangle\langle f_2|\bar{K}_0\rangle|^2 - |\langle f_1|\bar{K}_0\rangle\langle f_2|K_0\rangle|^2$$
$$-\langle f_1|K_0\rangle\langle f_2|\bar{K}_0\rangle\langle f_1|\bar{K}_0\rangle^*\langle f_2|K_0\rangle^* - \langle f_1|\bar{K}_0\rangle\langle f_2|K_0\rangle\langle f_1|K_0\rangle^*\langle f_2|\bar{K}_0\rangle^*) \tag{22}$$

where the terms in the first two lines of the equation come from $|A|^2$ and the rest comes from $|B|^2$ and the counter terms to cancel the terms added to make the desired expression of ζ. When $\zeta = 1$, the obvious interference terms [the terms proportional to $(1-\zeta)$] in the two probabilities expressed in the bases (K_S, K_L), eq.(19), and $(K^0, \bar{K}^0)$, eq.(21) respectively vanish. But the interference terms in the T_{extra} in the bases (K_S, K_L), eq.(21), the last line of T_{extra}, equation(22), are nonzero and become the maximum. The first two terms in equation (22) of T_{extra}

$$\frac{\zeta}{4}(|\langle f_1|K_0\rangle\langle f_2| K_0\rangle|^2 + |\langle f_1|\bar{K}_0\rangle\langle f_2|\bar{K}_0\rangle|^2) \tag{23}$$

cause $\Delta S = 2$ and $\Delta S = -2$ transitions respectively in the probability. We emphasize here that $|\Delta S| = 2$ transitions of order ζ could occur for $\zeta \neq 0$ even if the weak interactions has been turned off. Nonzero ζ breaks invariance of the probability under transformation of basis; namely the freedom of choice of the quantum axis is violated by a nonzero ζ. This may not sound strange because $\zeta \neq 0$ means after all that there exists a deviation from quantum mechanics upon which transformation invariance of basis is based upon. Therefore we could not disqualify on this ground equation (19) as a suggestive ways to deviate from the results obtained by quantum mechanics.

For a different experiment involving measurements on pairs of kaons in eigenstates of strangeness, $(K_0, \bar{K}_0)$, one may think to introduce a new decoherence parameter ζ' for $(K_0, \bar{K}_0)$ bases exactly in the same form for (K_S, K_L) bases as

$$P_{decoh}(f_1, f_2; t) = \frac{1}{2}[|\langle f_1|K_0\rangle\langle f_2|\bar{K}_0\rangle|^2 + |\langle f_1|\bar{K}_0\rangle\langle f_2|K_0\rangle|^2$$
$$-2(1 - \zeta')Re(\langle f_1|K_0\rangle\langle f_2|\bar{K}_0\rangle\langle f_1|\bar{K}_0\rangle^*\langle f_2|K_0\rangle^*)] \tag{24}$$

where $\zeta' = 0$ agrees to the result from quantum mechanics and $\zeta' = 1$ implies no interference term as before.

The relationship between the two decoherence parameters, ζ and ζ' can be obtained by equating (21) and (24). That is;

$$\zeta' = \zeta \cdot [1 + \frac{1}{2}\frac{T_{extra}}{Re(\langle f_1|K_0\rangle\langle f_2|\bar{K}_0\rangle\langle f_1|\bar{K}_0\rangle^*\langle f_2|K_0\rangle^*)}] \tag{25}$$

To see the magnitude of ζ', let us take $f_1 = \gamma(|K_0\rangle + \eta|\bar{K}_0\rangle)$ and $f_2 = \gamma(|\bar{K}_0\rangle - \eta|K_0\rangle)$ where η is a small number as an extreme example, and γ is a normalization. The denominator in equation(25) becomes as small as $\sim |\eta|^2$ while the numerator has a finite value, $\sim \frac{1}{2}$ meaning a large ζ'. This indicates that once ζ is nonzero, small deviation from quantum mechanics does not necessarily imply a small decoherence parameter in other basis ; more explicitly, an improper usage of the expression of equation(19), which is written for the analysis for the experiments involving measurements of eigenstates, K_S, K_L will cause a large decoherence parameter in agreement with the results of reference [5] in which the base-dependence of ζ is pointed out. Regardless, both decoherence parameters are severely constrained as shown in the next section and the expressions such as equation (19) and equation (24) can be used only for analysis on high precision measurements.

IV. UPPER BOUNDS FOR THE MAGNITUDES OF DECOHERENCE PARAMETERS

ζ induces nonzero probability for strangeness non-conserving, $\Delta S = \pm 2$ transitions of order ζ as shown in the first two terms in T_{extra} of equation(22) which is;

$$\frac{\zeta}{4}(|\langle f_1|K_0\rangle\langle f_2| K_0\rangle|^2 + |\langle f_1|\bar{K}_0\rangle\langle f_2|\bar{K}_0\rangle|^2) \tag{26}$$

This remains true at any arbitrary time t, as long as two sets of bases are related to each other by equations (8). To our best knowledge, the $|\Delta S| = 2$ transitions are limited by order of

the weak interaction. The $|\Delta S| = 2$ transition probability is proportional to $\Delta m(\sim 10^{-6} ev)$ which is the mass difference of K_L and K_S and can be estimated to be $\sim \frac{\Delta m}{m_c}$ using charm meson mass, m_c. Therefore the magnitudes of ζ is limited from the experiment as

$$\frac{\zeta}{4} \quad \leq \quad \sim \frac{\Delta m}{m_c} \quad \sim 10^{-12}$$

The corrections of weak interactions on the results may be estimated as follows: The eigenstates of effective hamiltonian

$$H_{ef} = H_{st} + H_{em} + H_{weak}$$

are give in equations (2, 3). Expanding the states in terms of ϵ, we obtain

$$|K_S\rangle = \frac{N}{\sqrt{2}}[(1 + \epsilon)|K^0\rangle - (1 - \epsilon)|\bar{K}^0\rangle] = \frac{1}{\sqrt{2}}[|K_S\rangle_{\epsilon=0} + \epsilon|K_L\rangle_{\epsilon=0} + O(\epsilon^2)]$$

$$|K_L\rangle = \frac{N}{\sqrt{2}}[(1 + \epsilon)|K^0\rangle + (1 - \epsilon)|\bar{K}^0\rangle] = \frac{1}{\sqrt{2}}[|K_L\rangle_{\epsilon=0} + \epsilon|K_S\rangle_{\epsilon=0} + O(\epsilon^2)] \quad (27)$$

Therefore the 1^{--} state of two neutral kaons can be written in terms of CP eigenstates as

$$(|K_S(t)\rangle|K_L(t)\rangle - |K_L(t)\rangle|K_S(t)\rangle) = (|K_S(t)\rangle_{\epsilon=0}|K_L(t)\rangle_{\epsilon=0} - |K_L(t)\rangle_{\epsilon=0}|K_S(t)\rangle_{\epsilon=0})$$
$$+ O(\epsilon^2) terms \quad (28)$$

where $O(\epsilon^2) terms$ represents the sum of products of combinations of $|K_S(t)\rangle_{\epsilon=0}$ and $|K_L(t)\rangle_{\epsilon=0}$ of order of $O(\epsilon^2)$ and higher. Among them, there will be terms such as $|K_S(t)\rangle_{\epsilon=0}|K_S(t)\rangle_{\epsilon=0}$, and $|K_L(t)\rangle_{\epsilon=0}|K_L(t)\rangle_{\epsilon=0}$ which are CP violating, CP=+, states . Note there is no correction of order ϵ and the corrections to the equations from (19) to (22) are order of ϵ^2 which can be neglected when we are taking effects of order ϵ into accounts. Therefore the upper limits for the magnitudes of decoherence parameters for the basis of effective hamiltonian (K_s, K_L) of equation (1)is

$$\zeta \quad \leq \sim O(|\epsilon|^2)$$

Therefore we can safely state

$$\frac{\zeta}{4} \quad \leq \quad |\epsilon|$$

Similarly, if equation (24) is used for analysis on data from measurements of strangeness eigenstates, we inevitably end up introducing CP violating transitions terms of order ζ';

$$\frac{\zeta'}{4}(|\langle f_1|K_S\rangle\langle f_2| K_S\rangle|^2 + |\langle f_1|K_L\rangle\langle f_2|K_L\rangle|^2)$$

The derivation of this two terms is trivial because the mathematical procedures are exactly reversed but identical for the transformation from the strangeness basis $(K^0, \bar{K}^0)$ to CP basis $(K_S, K_L)_\epsilon = 0$. As CP violating transitions are bounded by order of ϵ, we get

$$\frac{\zeta'}{4} \quad \leq \quad |\epsilon| \quad (29)$$

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Therefore conservatively the coherence parameters are limited by

$$\zeta', \quad \zeta \quad \leq \quad 4|\epsilon| \tag{30}$$

The stringent limit on the decoherence parameters limits the usage of equations such as (19) and (24) in the data analysis. Inversely, a large decoherence parameter is a signal for an erroneous use of formulation or mistakes made in analysis.

The situation discussed above can be understood more clearly when we take an example in the more familiar case of spin $\frac{1}{2}$ particles, say two correlated electrons with total spin 0. The choice of quantization axis, can be chosen from any one of axis, x, y, or z. It is free for user's choice and the contents of physics must be independent of the choice one makes. Total angular momentum is conserved and there has been no violation of this conservation observed so far. We imagine a situation in which two electrons are moving apart on y axis in opposite direction. Two sets of bases may be in z and x axis and their spin states are related by

$$|x \uparrow\rangle = \frac{1}{\sqrt{2}}(|z \uparrow\rangle - |z \downarrow\rangle)$$

$$|x \downarrow\rangle = \frac{1}{\sqrt{2}}(|z \uparrow\rangle + |z \downarrow\rangle) \tag{31}$$

which are analogous to equation (8): If we make substitutions

$$|x \uparrow\rangle \rightleftharpoons |K_S\rangle$$
$$|x \downarrow\rangle \rightleftharpoons |K_L\rangle$$
$$|z \uparrow\rangle \rightleftharpoons |K^0\rangle$$
$$|z \downarrow\rangle \rightleftharpoons |\bar{K}^0\rangle \tag{32}$$

the analogy is clear. There is no time-dependence in the probability unless we choose a time dependent decoherence parameter. The probability for finding one of electron spin up and the other down is

$$P_{spin}(f_1, f_2; t) = \frac{1}{2}[|\langle f_1|x \uparrow\rangle\langle f_2|x \downarrow\rangle|^2 + |\langle f_1|x \downarrow\rangle\langle f_2|x \uparrow\rangle|^2$$
$$-(1 - \xi)(\langle f_1|x \uparrow\rangle\langle f_2|x \uparrow\rangle\langle f_1|x \downarrow\rangle^*\langle f_2|x \downarrow\rangle^*$$
$$+\langle f_1|x \uparrow\rangle^*\langle f_2|x \uparrow\rangle^*\langle f_1|x \downarrow\rangle\langle f_2|x \downarrow\rangle)] \tag{33}$$

where we introduced a decoherence parameter ξ and $\xi = 0$ gives the prediction of probability for two electron's spins, f_1 and f_2 in quantum mechanics. By four substitutions in equation (32), we see that at $\xi = 0$ the probability is invariant under base transformations ; namely changing x quantization axis to z quantization axis, we can show by substitutions that the form of $P_{spin}(f_1, f_2; t)$ in z-basis does not change. For none zero ξ, by going through the same calculation performed for kaons or again by substitutions of (32), we obtain

$$P_{spin}(f_1, f_2; t) = \frac{1}{2}[|\langle f_1|z \uparrow\rangle\langle f_2|z \downarrow\rangle|^2 + |\langle f_1|z \downarrow\rangle\langle f_2|z \uparrow\rangle|^2$$
$$-(1 - \xi)(\langle f_1|z \uparrow\rangle\langle f_2|z \uparrow\rangle\langle f_1|z \downarrow\rangle^*\langle f_2|z \downarrow\rangle^*$$
$$+\langle f_1|z \uparrow\rangle^*\langle f_2|z \uparrow\rangle^*\langle f_1|z \downarrow\rangle\langle f_2|z \downarrow\rangle)] + \xi t_{extra} \tag{34}$$

where t_{extra} is

$$\frac{1}{2}(|\langle f_1|z\uparrow\rangle\langle f_2|z\uparrow\rangle|^2 + |\langle f_1|z\downarrow\rangle\langle f_2|z\downarrow\rangle|^2$$
$$-\langle f_1|z\uparrow\rangle\langle f_2|z\uparrow\rangle\langle f_1|z\downarrow\rangle^*\langle f_2|z\downarrow\rangle^* - \langle f_1|z\uparrow\rangle^*\langle f_2|z\uparrow\rangle^*\langle f_1|z\downarrow\rangle\langle f_2|z\downarrow\rangle$$
$$-|\langle f_1|z\uparrow\rangle\langle f_2|z\downarrow\rangle|^2 - \langle f_1|z\downarrow\rangle\langle f_2|z\uparrow\rangle|^2$$
$$-\langle f_1|z\uparrow\rangle\langle f_2|z\downarrow\rangle\langle f_1|z\downarrow\rangle^*\langle f_2|z\uparrow\rangle^* - \langle f_1|z\downarrow\rangle\langle f_2|z\uparrow\rangle\langle f_1|z\downarrow\rangle^*\langle f_2|z\uparrow\rangle^*) \tag{35}$$

In the first two terms in t_{extra}, the transformation to z quantization axis induced a probability for finding two electrons spin up of order of ξ. In this case these two terms violates total angular momentum conservation. And therefore the decoherence parameter ξ for two spin correlated electrons has to be zero. Otherwise not only the foundations of quantum mechanics such as equation (31), becomes not true but also uniformity of space will be lost as soon as $\xi \neq 0$ takes place. The decoherence or separation of two correlated spin $\frac{1}{2}$ particles in the form of equation (33) could not occur [9] without violating the well established laws of physics.

V. CONCLUDING REMARKS

Our naive intuition cultivated from interference phenomena of photons and electrons tells us that the interference terms in probabilities are one of the typical quantum mechanical quantities and it is natural to think that a break down of quantum mechanics might be accounted by changing the interference terms. However that is not so for the correlated two neutral kaon case: Because even if the interference term is zero in a given bases, it could be large in another bases as seen in the example given in section 2. In other words, even if the probability of a system is described only by a sum of probabilities (no interference terms), it dose not imply that the system is "more classical" or separable or disentangled. Another example for which our cultivated intuition for quantum mechanics doesn't work is exhibited [10] in neutrino oscillations: Our naive intuition predicts that the smaller the mass difference(the gap of two energy levels) , the larger the rate of transition between them. However it is the other way around; we observe easily (transition happens more often) for the larger mass difference as the probability of finding ν_l types neutrino in a beam which is initially pure ν neutrino, as function of time, is given by

$$P(\nu \rightarrow \nu_l) \sim sin^2(\frac{\Delta ML}{4E}) \qquad (\nu \neq \nu_l) \tag{36}$$

where $\Delta M^2 = m_\nu^2 - m_{\nu_l}^2$ and E and L are the energy and traveled distance of ν.

We have shown that the decoherence in correlated two neutral kaons in the form of modified interference terms is limited ; equation(24) with a condition $|\zeta'| \leq 10^{-2}$ and equation(1) with a condition $|\zeta| \leq 10^{-5}$ may be used to analyze experimental data involving measurement of two neutral kaons in strangeness states, $(K^0, \bar{K}^0)$ and in mass eigenstates, (K_S, K_L) respectively.

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